Thoughts About Power and Effect Size in Multilevel Analysis

Full paper available at:

http://www.hlm-online.com/papers/

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The Problem

- Determining the proportion of the total variance that lies systematically between schools, called the intraclass correlation (ICC), constitutes the first step in an HLM analysis. We conduct this analysis with a fully unconditional model, which means that no student or school characteristics are considered. This first step can also indicate whether HLM is needed or whether a single level analytic method is appropriate. *Only when the ICC is more than trivial* (i.e., greater than 10% of the total variance in the outcome) *would the analyst need to consider multilevel methods* [emphasis mine]. Ignoring this step (i.e., assuming an ICC of either 0 or 1) would be inappropriate if the research question were multilevel. Investigation of contextual effects, I argue, is by nature a multilevel question. (Lee, 2000, p. 128).

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The Tested Models

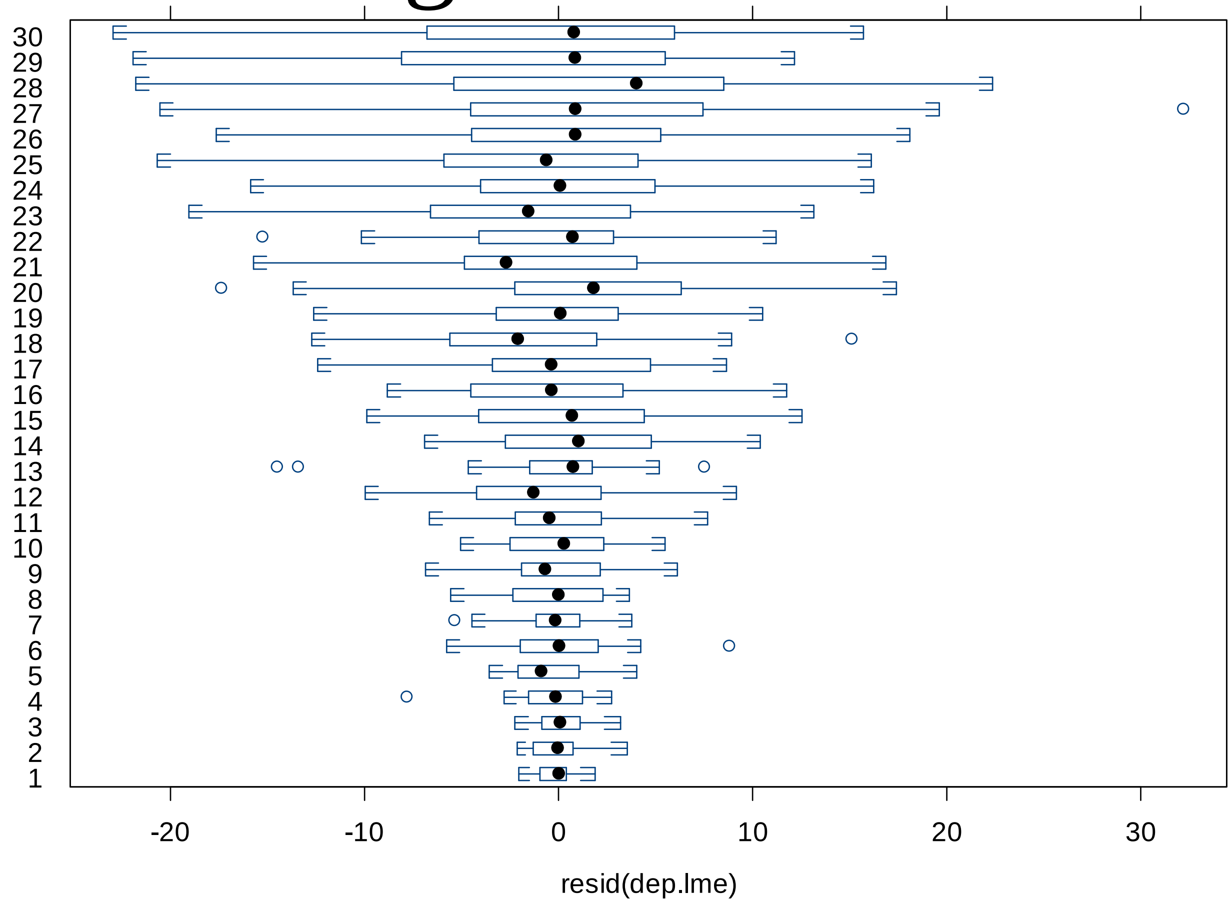
M0 - $dv_{ij} = \gamma_{00} + u_{0j} + e_{ij}$

M1 - $dv_{ij} = \gamma_{00} + \gamma_{10}(iv_{ij}) + u_{0j} + u_{1j}(iv_{ij}) + e_{ij}$

Fitted Estimates

	M_0: Null model		M_1: + dv & random	
Fixed Effects:	estimate	s.e.	estimate	s.e.
Intercept γ_{00}	49.89	0.21	40.56	0.91
Slope γ_{10}			-0.01	0.13
Random Effects:				
Level-1 effect, σ^2_{e0}	38.02		2.48	
Intercept, σ^2_{u0}	$<$ .00001		24.23	
Slope, σ^2_{u1}			0.48	
$COV(u_0, u_1)$,			0.02	
Fit:				
X^2	5828.70		3724.25	
AIC	5834.70		3736.25	
BIC	5849.11		3765.05	

The First Example
Breaking all of the Rules!!



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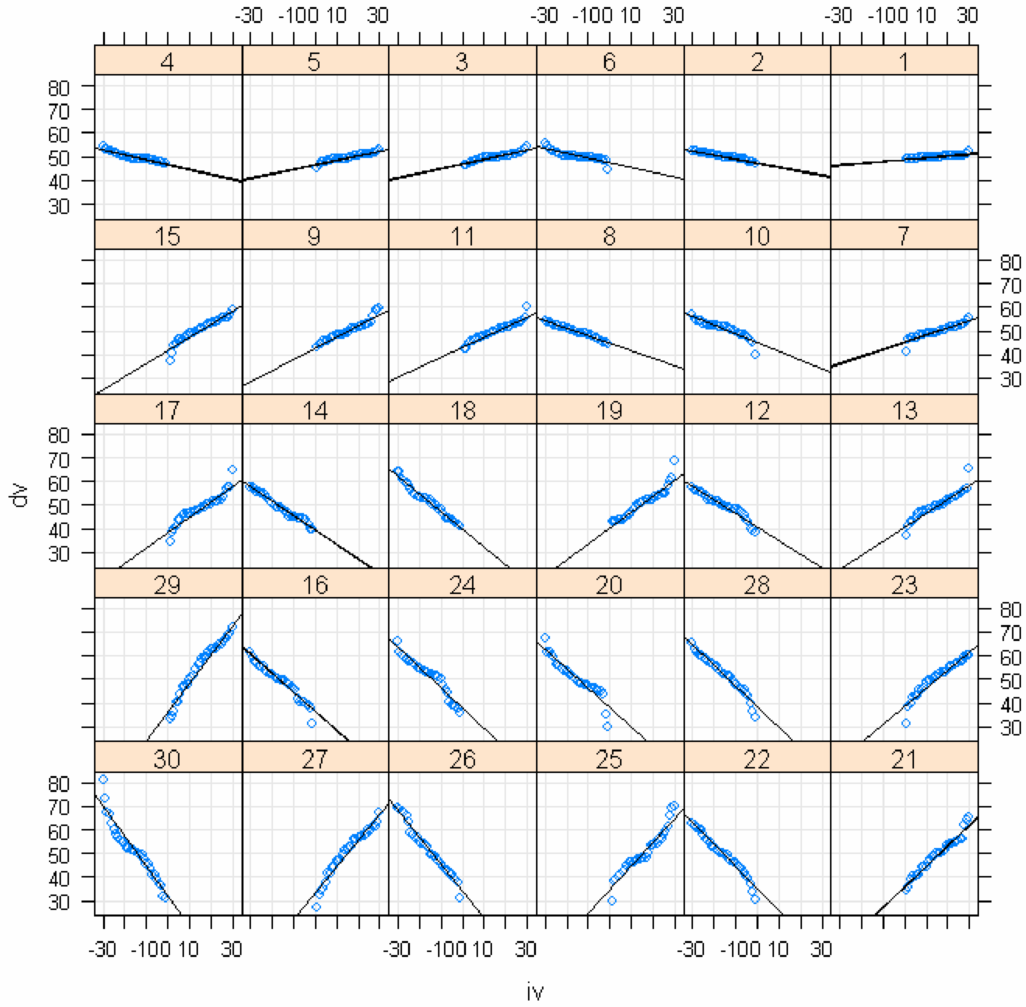
Fitted Trajectories



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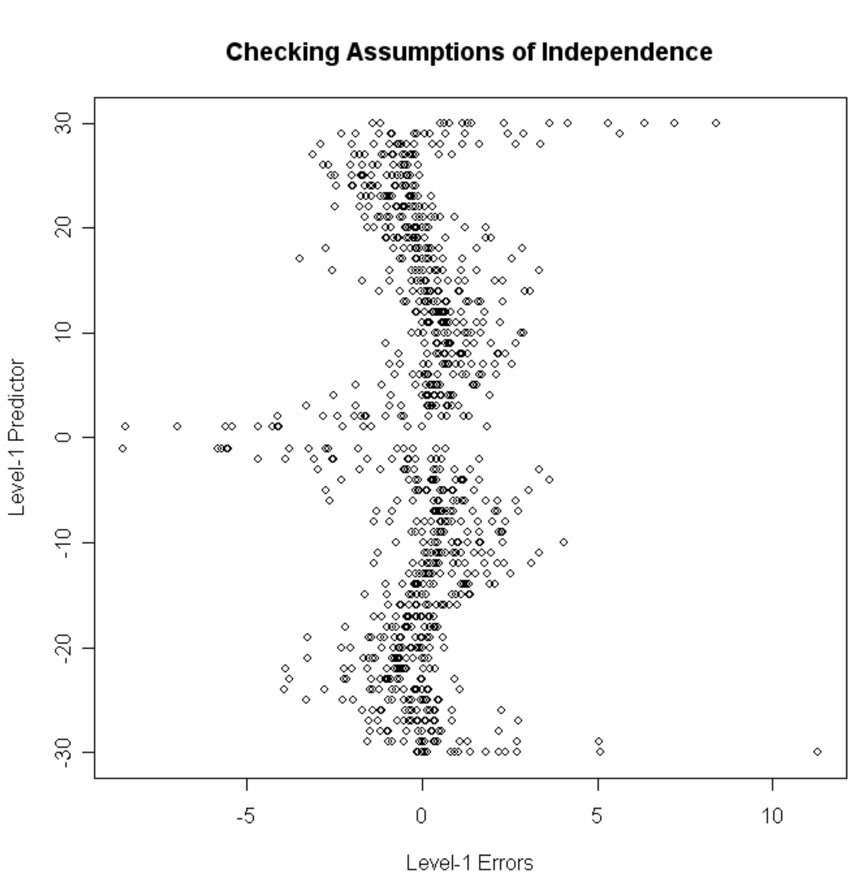
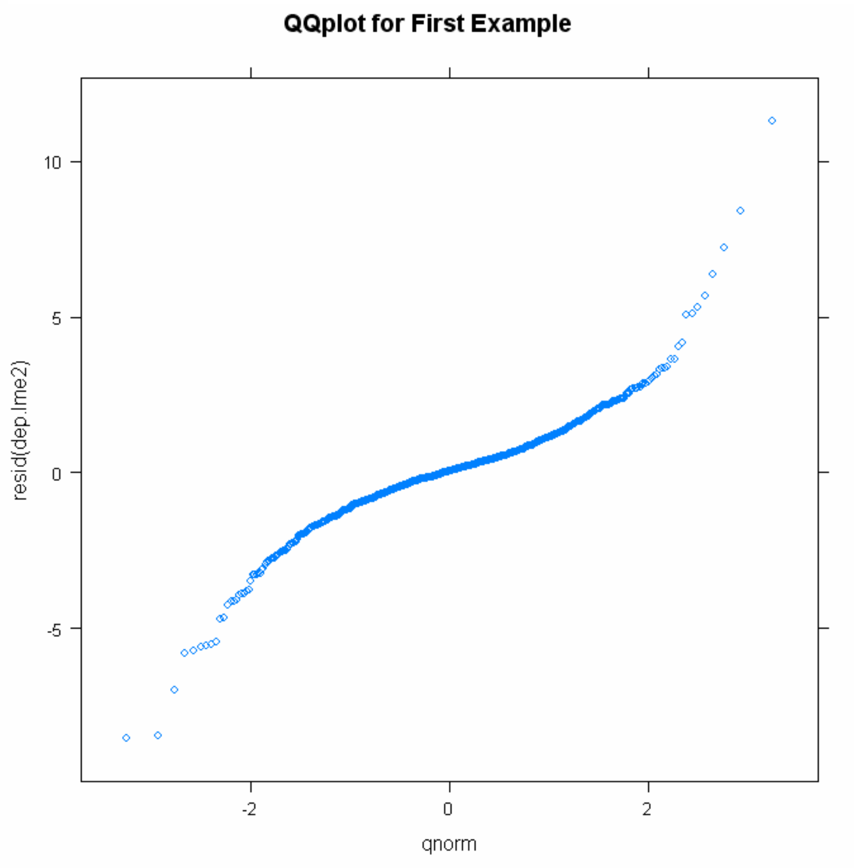
More Assumption Problems





HLM Effect Size

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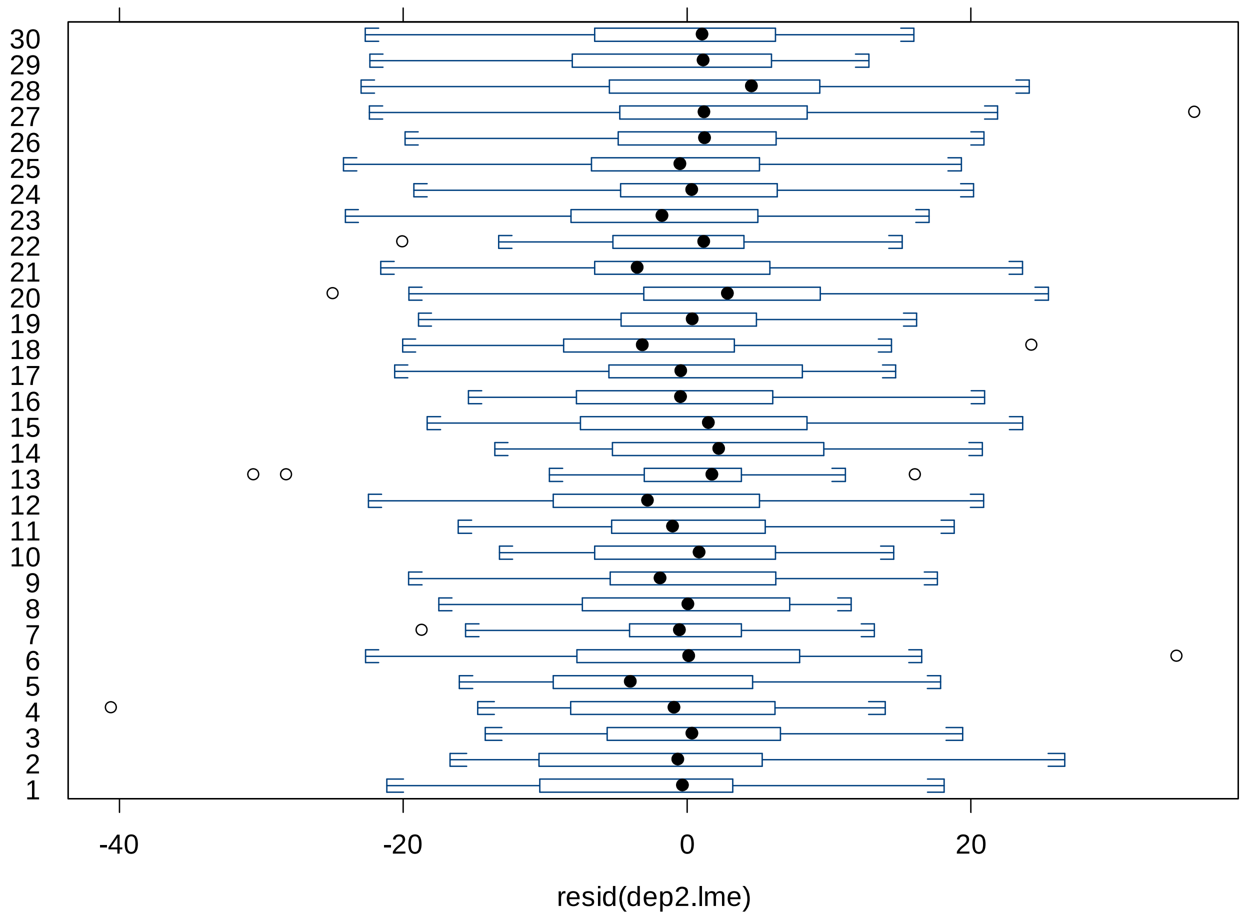
The Second Example



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Fitted Estimates

	M_0: Null model		M_1: + dv & random	
Fixed Effects:	estimate	s.e.	estimate	s.e.
Intercept γ_{00}	49.61	0.33	-74.32	18.16
Slope γ_{10}			2.47	0.36
Random Effects:				
Level-1 effect, σ^2_{e0}	98.29		8.01	
Intercept, σ^2_{u0}	$>$ .00001		9821.94	
Slope, σ^2_{u1}			3.87	
$COV(u_0, u_1)$,			-195.04	
Fit:				
X^2	6682.62		4683.29	
AIC	6688.62		4695.29	
BIC	6703.03		4724.09	

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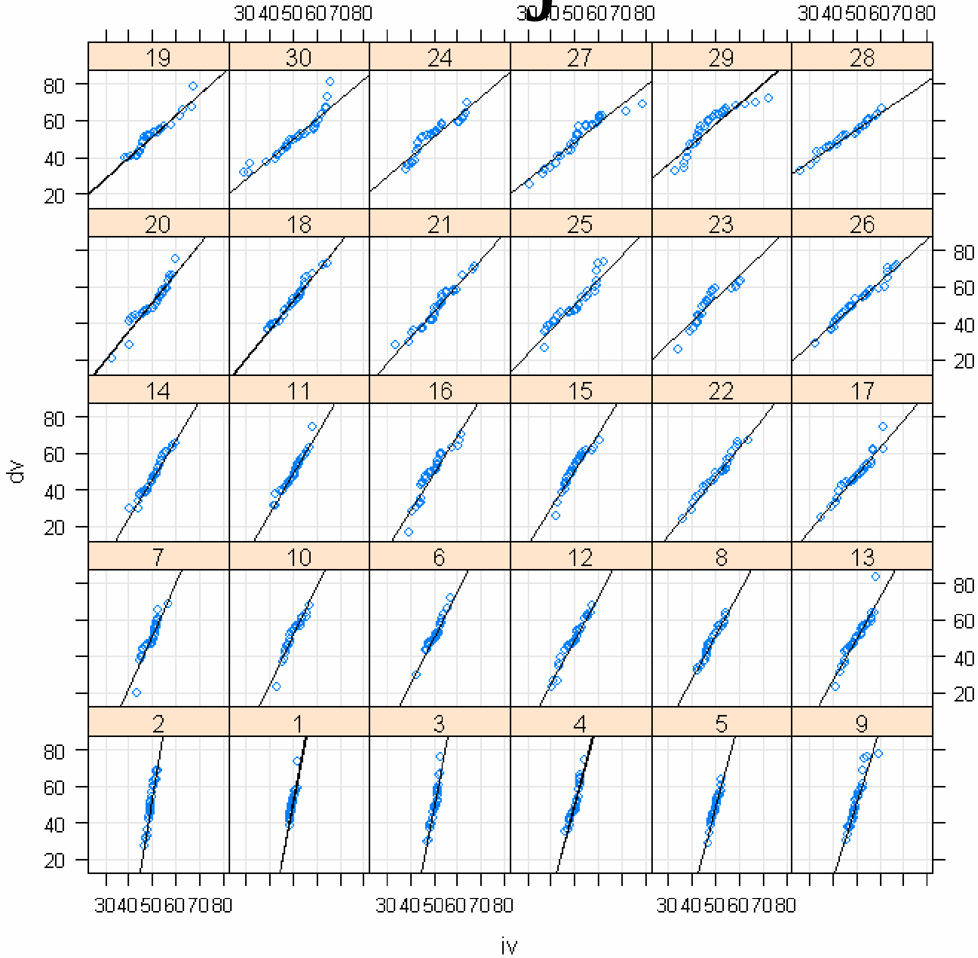
Fitted Trajectories



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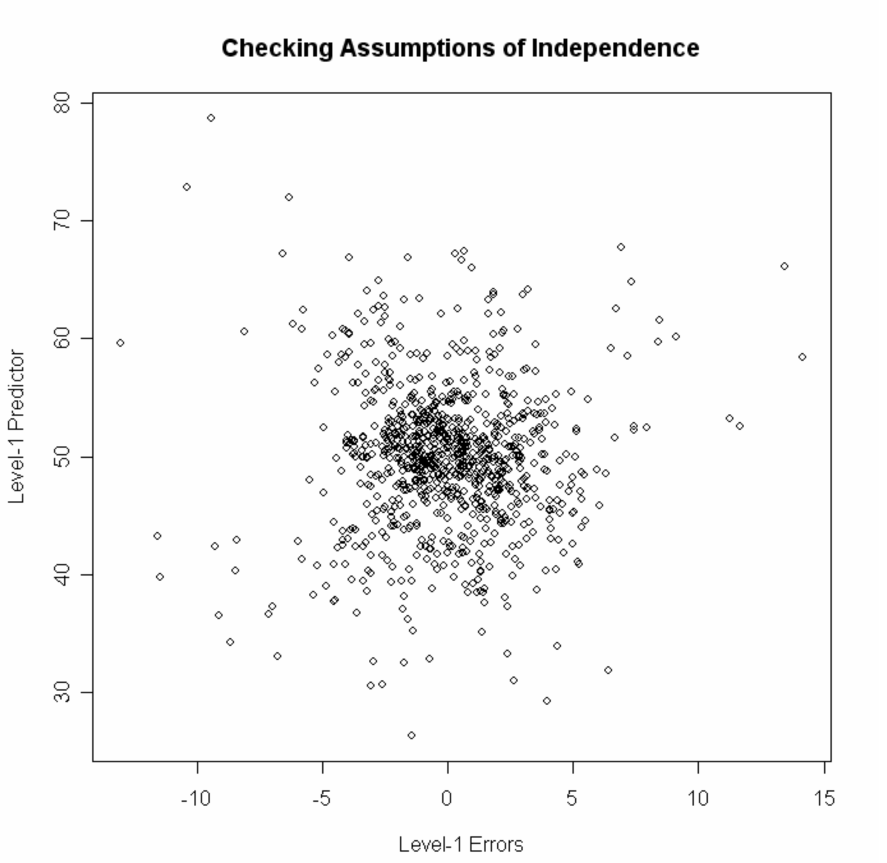
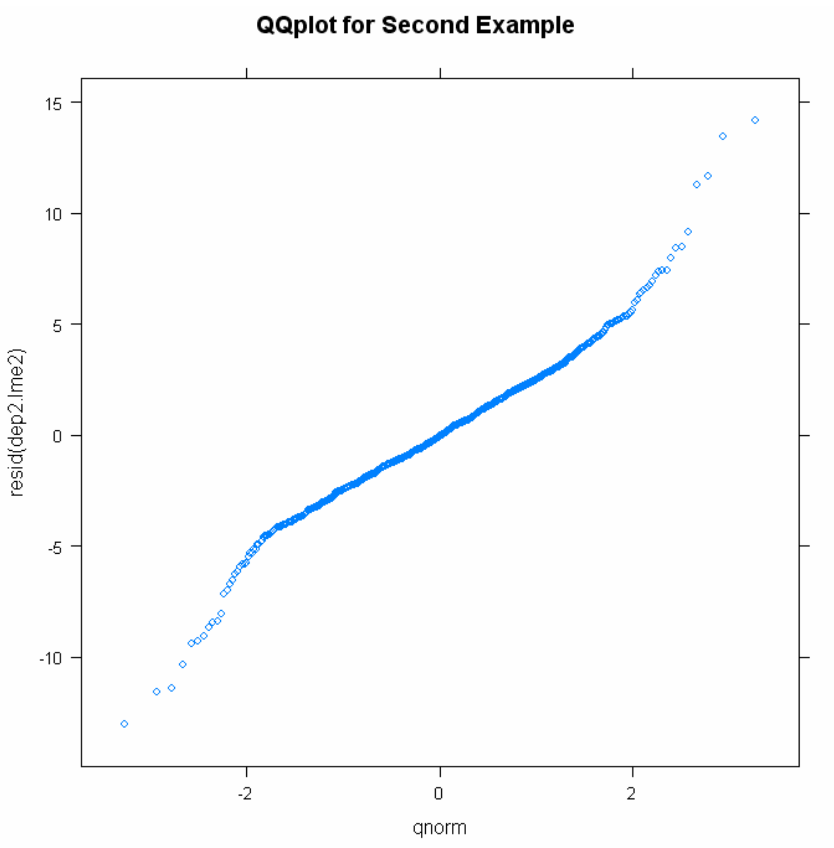
Checking Assumptions





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Previous Ideas for Effect Sizes in HLM

- Null model versus Full model measures

$$R_1^2 = \frac{\sigma_{e|b}^2 - \sigma_{e|m}^2}{\sigma_{e|b}^2}$$

$$R_2^2 = \frac{\sigma_{u0|b}^2 - \sigma_{u0|m}^2}{\sigma_{u0|b}^2}$$

Explained Variance as a Reduction in Mean Square Prediction Error

- Snijders and Bosker (1999)

$$R_1^2 = 1 - \frac{\mathrm{var}\left(Y_{ij} - \sum_h \gamma_h X_{hij}\right)}{\mathrm{var}\left(Y_{ij}\right)} = 1 - \frac{\hat{\sigma}^2(full) + \hat{\tau}_0^2(full)}{\hat{\sigma}^2(null) + \hat{\tau}_0^2(null)}$$

$$R_2^2 = 1 - \frac{\mathrm{var}\left(\overline{Y}_{.j} - \sum_h \gamma_h \overline{X}_{h.j}\right)}{\mathrm{var}\left(\overline{Y}_{.j}\right)} = 1 - \frac{\frac{\hat{\sigma}^2(full)}{B} + \hat{\tau}_0^2(full)}{\frac{\hat{\sigma}^2(null)}{B} + \hat{\tau}_0^2(null)}$$

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Problems with these Measures

- It is possible to obtain “negative” values for variance explained in these models
- For example, if we add a group-level predictor, then we could expect that it would reduce only the between-groups variance, not the within-groups variance, ultimately increasing the estimate for the population variance.

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Distance Measure for R^2 in HLM

- Remember in multiple regression that:

$$R^2_{y(x1,x2,x3)} = R^2_{y\hat{y}}$$

- In HLM we can define the SS_{total}

$$\sigma'^2_{total} = \sum_{j=1}^{J} \sum_{i=1}^{n_j} \left(y_{ij} - \tilde{y}_j\right)^2 = e^2_{ij}$$

- where

$$\tilde{y}_{ij} = \gamma_{00}(cons) + u_{0j}$$

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Distance Measure (cont)

- The SS_{error} would be thought of as:

$$\sigma'^{2}_{error} = \sum_{j=1}^{J} \sum_{i=1}^{n_j} \left(y_{ij} - \hat{y}_{ij} \right)^2$$

- where

$$\hat{y}_{ij} = \gamma_{00} + \gamma_{q0} X_{ij} + \gamma_{0q} W_j + u_{0j} + e_{ij}$$

- Then it follows that:

$$1 - \frac{\sigma'^{2}_{error}}{\sigma'^{2}_{total}}$$

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R^2 Measure that Incorporates a Gaussian Probability Density Function

- By applying the Gaussian pdf, we can define

$$\sigma^2_{total} = \frac{\sum_{ij} p(y_{ij})(e'_{ij})^2}{\sum_{ij} p(y_{ij})}$$

- where

$$p(y_{ij}) = p(d_{ij} \mid s_j)\, p(s_j)$$

- and

$$p(y_{ij}) = \frac{1}{\sqrt{2\pi\sigma^2_{ij} \cdot \sigma^2_j}} \bullet \exp\left[-\frac{(e'_{ij})^2}{2\sigma^2_{ij}}\right] \bullet \exp\left[-\frac{(u'_j)^2}{2\sigma^2_j}\right]$$

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R^2 Measure that Incorporates a Gaussian Probability Density Function (cont)

- We can also compute the error term as

$$\sigma^2_{error} = \frac{\sum_{ij} p(y_{ij})(e''_{ij})^2}{\sum_{ij} p(y_{ij})}$$

- where

$$p(y_{ij}) = \frac{1}{\sqrt{2\pi\sigma^2_{ij} \cdot \sigma^2_j}} \bullet \exp\left[-\frac{(e''_{ij})^2}{2\sigma^2_{ij}}\right] \bullet \exp\left[-\frac{(u''_j)^2}{2\sigma^2_j}\right]$$

- Then finally we have:

$$1 - \frac{\sigma^2_{error}}{\sigma^2_{total}}$$

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Group Initiated R^2 Based on Weighted Least Squares

- γ_{00} is estimated by $\hat{\gamma}_{00} = \sum \Delta_j^{-1} \overline{Y}_{\bullet j} / \sum \Delta_j^{-1}$ where Δ_j is a precision parameter
- The effect of these precision parameters on the grand estimate is to apply more weight to the groups that are measured with more precision

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Building from OLS

- In OLS, the R^2 can be thought of as

$$R^2 = 1 - \frac{\sum_{i=1}^{n} \left(\hat{y}_i - y_i\right)^2}{\sum_{i=1}^{n} \left(y_i - \bar{y}\right)^2}$$

- This is the mathematical equivalent of the following (for a single group)

$$R_j^2 = 1 - \frac{1}{n_j} \sum_{i=1}^{n_j} \frac{\left(\hat{y}_{ij} - y_{ij}\right)^2}{\sigma_{ij}^2}$$

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WLS R^2

- For simplification purposes, we define

$$R_j^2 = 1 - \frac{1}{n_j}\sum_{i=1}^{n_j}\frac{\left(\hat{y}_{ij} - y_{ij}\right)^2}{\sigma_{ij}^2} = 1 - \frac{1}{n_j}\sum_{i=1}^{n_j} E_i$$

- This would mean that the total weighted least squares normalized error for all groups could be thought of as:

$$E_j = \sum_{j=1}^{J} p\left(s_j\right)E_i \quad \text{where} \quad p\left(s_j\right) = \frac{1}{\sqrt{2\pi\sigma_j^2}} \bullet \exp\left[-\frac{\left(u_j'\right)^2}{2\sigma_j^2}\right]$$

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WLS R^2 (cont)

- We then expand the previous equation to include all groups and also reflect the need to use a weighted estimator

$$R_T^2 = 1 - \frac{1}{\sum_{j=1}^{J} p(s_j) n_j} E_j$$

- After solving for the above equation (see handout) we get:

$$R_T^2 = \frac{\sum_{j=1}^{J} n_j \cdot p(s_j) \cdot R_j^2}{\sum_{j=1}^{J} n_j \cdot p(s_j)}$$

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Does it Work????

- An example from Roberts (2004) available at http://www.hlm-online.com/datasets/
- This can be easily computed in R.

HLM-Online.com

Roberts (2004) Data

$$science_{ij} = \gamma_{00} + u_{0j} + (\gamma_{10} + u_{1j})urban + e_{ij}$$

	M$_0$: Null model		M$_1$: + Urbanicity		M$_2$: + random est.	
Fixed Effects:	estimate	s.e.	estimate	s.e.	estimate	s.e.
Intercept	10.688	1.268	22.303	2.426	22.391	2.717
Urbanicity			-0.805	0.048	-0.867	0.130
Random Effects:						
σ^2_{ε}	1.979		0.655		0.271	
σ^2_{u0}	25.531		86.456		113.603	
σ^2_{u1}					0.253	
σ_{u01}					-3.344	
Fit:						
X^2	637.856		500.094		412.171	
AIC	643.856		508.094		424.171	
BIC	653.063		520.344		442.547	

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R^2 in HLM

Level-1 Equation

$$R_1^2 = \frac{\sigma_{e|b}^2 - \sigma_{e|m}^2}{\sigma_{e|b}^2}$$

$$R_1^2 = \frac{1.979 - 0.271}{1.979} = .863$$

Level-2 Equation

$$R_2^2 = \frac{\sigma_{u0|b}^2 - \sigma_{u0|m}^2}{\sigma_{u0|b}^2}$$

$$R_2^2 = \frac{25.531 - 86.456}{25.531} = -2.386$$

??? -238% Variance Explained???

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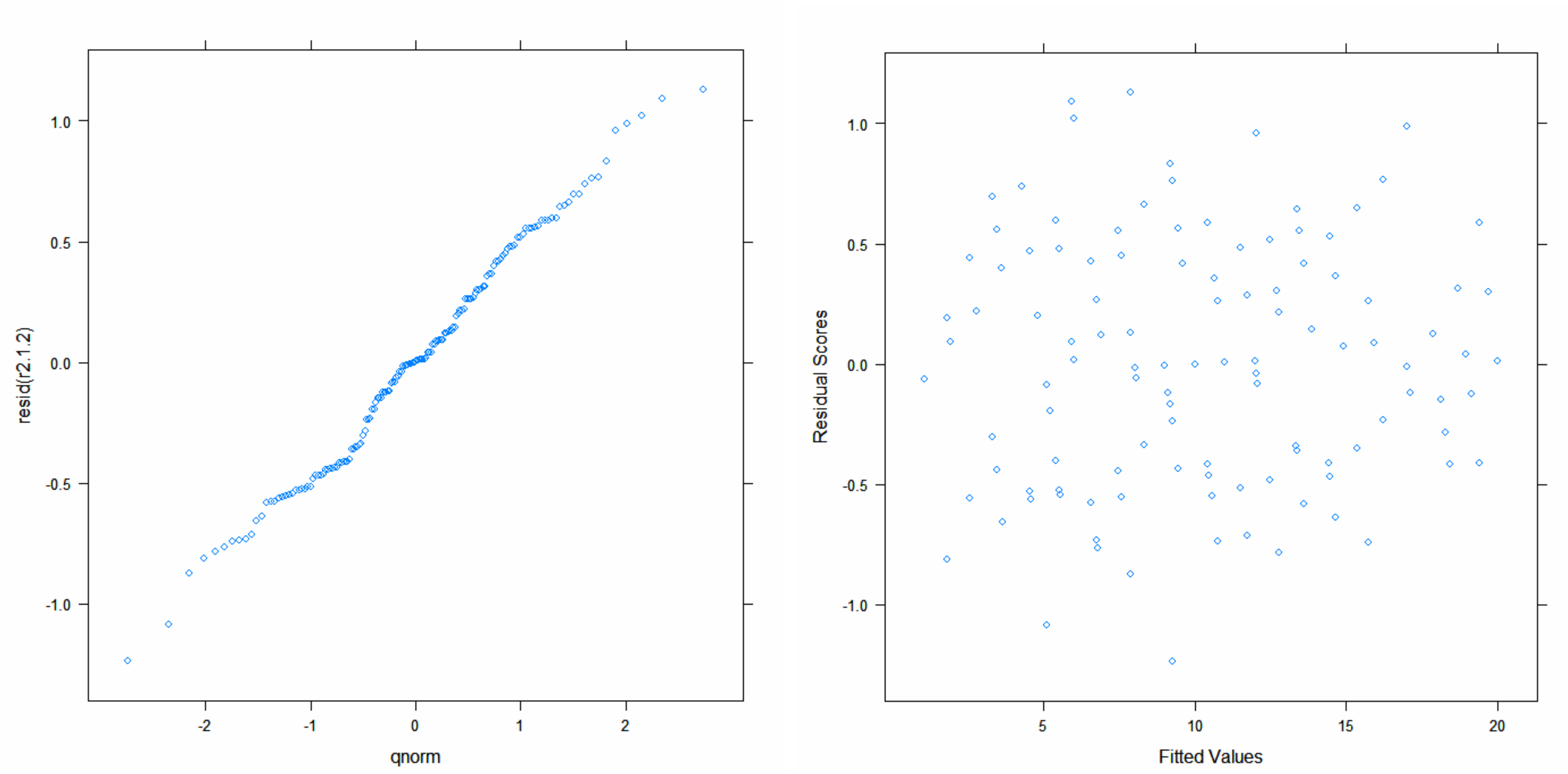
Is the Model Misspecified?



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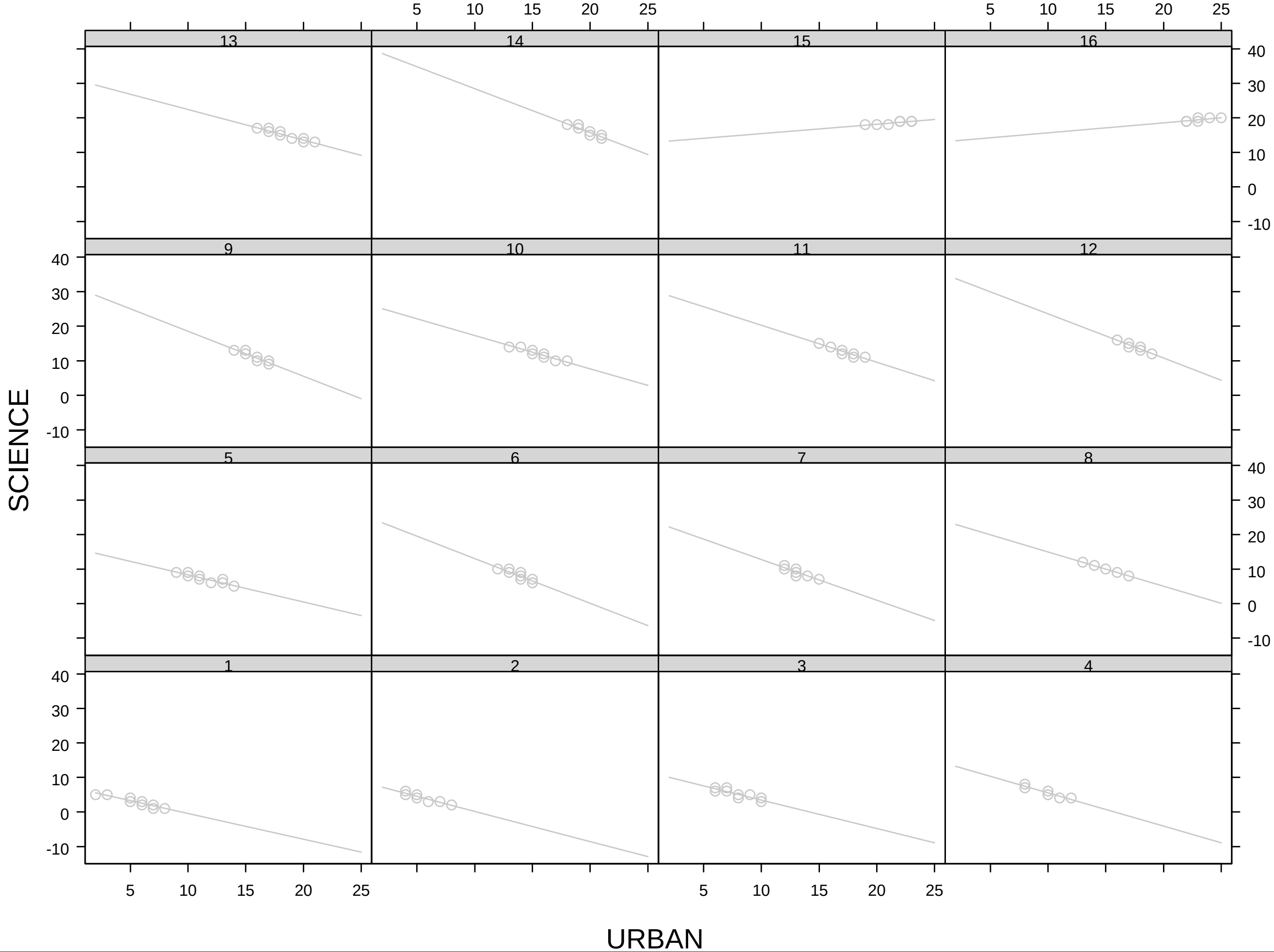
Roberts (2004) Data (cont.)



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Computing the R^2

$$R_T^2 = \frac{\sum_{j=1}^{J} n_j \cdot p(s_j) \cdot R_j^2}{\sum_{j=1}^{J} n_j \cdot p(s_j)}$$

$$R_T^2 = 0.865$$

Group	Intercept	Slope	R_j^2	$p(s_j)$
1	7.04	-0.747	0.87	0.013
2	8.90	-0.874	0.89	0.017
3	11.67	-0.822	0.79	0.023
4	15.13	-0.961	0.87	0.030
5	16.19	-0.785	0.86	0.032
6	26.03	-1.297	0.81	0.035
7	24.55	-1.178	0.84	0.037
8	24.89	-0.993	1.00	0.036
9	31.57	-1.302	0.88	0.026
10	26.97	-0.966	0.88	0.034
11	30.98	-1.071	0.93	0.027
12	36.36	-1.278	0.93	0.016
13	31.27	-0.884	0.89	0.026
14	41.20	-1.273	0.88	0.008
15	12.72	0.271	0.84	0.025
16	12.79	0.278	0.67	0.025

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